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PARAMETER IDENTIFICATION FOR A COMPLEX LEAD–ACID BATTERY MODEL BY COMBINING FUZZY CONTROL AND STOCHASTIC OPTIMIZATION

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ABSTRACT

The accurate simulation of lead–acid batteries requires the use of sophisticated models based on first principles containing many parameters. Existing methods for parameter identification often fail due to many local minima of the error function or the high computational needs to cover the parameter space. Therefore a novel approach for parameter identification with complex physical models containing many unknown parameters is presented. It is based on the utilization of available expert knowledge regarding specific model parameters. The expert knowledge is integrated through fuzzy control and combined with stochastic optimization algorithms for solving the battery identification problem.

INTRODUCTION

The accurate simulation of lead–acid battery cells is of growing interest in the automotive industry, especially in hybrid and electric vehicle technology. The battery is a fundamental component of the powertrain of such vehicles. Its nonlinear behavior under various operating conditions has to be considered for the implementation of new drive concepts. Various models for lead–acid batteries exist which are based on a detailed description of the electrochemical, thermal and transport processes in the battery [1–5]. The models consist of systems of partial differential equations that are highly nonlinear and depend on several different parameters. Many of them can only be measured with elaborate, often destructive methods, while some cannot be measured at all. The only quantities that are easily accessible are the cell voltage and the cell current. There is an increasing need for a fast parameterization of the battery models at low cost since the parameters of batteries from different manufacturers show considerable differences. In addition, the values depend on the state of health of the battery. They are slowly changing with time. This long–time behavior of the batteries cannot be predicted by present models. A frequent re–parameterization is therefore necessary.

Classical approaches to parameter identification very often fail to satisfy the demands for accuracy and speed. A new approach for parameter identification of batteries was therefore developed. It tries to utilize all available knowledge about the model parameters to achieve a better performance. The method is solely based on the measurements of cell current and cell voltage. These quantities can be obtained very easily and with low cost. The available expert knowledge is built into a fuzzy controller [6]. The resulting control loop is embedded in a Genetic Algorithm (GA) [7].

The following section gives an introduction to the used lead–acid battery model. After that the novel parameter identification method is described in detail, including the accumulation of expert knowledge, the fuzzy control loop and the GA. The identification results for a real battery are afterwards presented, followed by some concluding remarks on the presented identification method and future developments.

LEAD–ACID BATTERY MODEL

The used battery model (based on [1–5]) describes a single lead–acid battery cell with starved electrolyte. Originated on electrical, chemical, thermal, physical and material transport phenomena the formulation is based on a macroscopic de-

scription of porous electrodes. The cell consists of a porous PbO_2 electrode with conductivity σ_1, a porous Pb electrode with conductivity σ_3 and a porous non–conducting separator in between (figure 1).

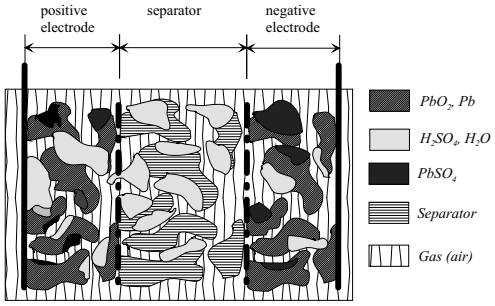

Figure 1: Cross–section of the cell model: Porous PbO_2 (left, region 1) and Pb (right, region 3) electrodes; a porous separator (middle, region 2)

Acid with conductivity κ is present in each of the regions. For the model it is a reasonable simplification to consider space coordinates in only one dimension (thus all quantities are related to the geometrical area of the electrodes).

The amount of the materials, which is a function of space and time are described by volume fractions for the solid and for the liquid phase (ε_s and ε_l):

$$\frac{\partial \varepsilon_\mathrm{s}}{\partial t} = \mathrm{K}_1 j_\mathrm{main} \tag{1}$$

$$\frac{\partial \varepsilon_\mathrm{l}}{\partial t} = \mathrm{K}_2 j_\mathrm{main} + \mathrm{K}_3 j_{O_2} + \mathrm{K}_4 j_{\mathrm{H}_2} \tag{2}$$

j_main, j_{O_2} and j_{H_2} are the current densities due to the main charge/discharge-, oxygen- and hydrogen reaction which are defined separately for region 1 and 3 labelled by an additional subscript, as:

$$j_{\mathrm{main},1} = a_\mathrm{max} i_0 \left(1-\mathcal{U}\right)^\zeta \cdot$$

$$\frac{\mathrm{L}_1 a_\mathrm{max} k_{\mathrm{PbSo}_4} \mathcal{U}^\zeta \left[1 - e^{-(\alpha_\mathrm{a}+\alpha_\mathrm{c})\mathrm{L}_2\eta}\right]}{a_\mathrm{max} i_0 \left(1-\mathcal{U}\right)^\zeta + \mathrm{L}_1 a_\mathrm{max} k_{\mathrm{PbSo}_4} \mathcal{U}^\zeta e^{-\alpha_\mathrm{a}\mathrm{L}_2\eta}} \tag{3}$$

$$j_{\mathrm{main},3} = a_\mathrm{max} i_0 \left(1-\mathcal{U}\right)^\zeta \cdot$$

$$\frac{j_{\mathrm{O}_2} + \mathrm{L}_1 a_\mathrm{max} k_{\mathrm{PbSo}_4} \mathcal{U}^\zeta \left[e^{(\alpha_\mathrm{a}+\alpha_\mathrm{c})\mathrm{L}_2\eta} - 1\right]}{a_\mathrm{max} i_0 \left(1-\mathcal{U}\right)^\zeta + \mathrm{L}_1 a_\mathrm{max} k_{\mathrm{PbSo}_4} \mathcal{U}^\zeta e^{\alpha_\mathrm{c}\mathrm{L}_2\eta}} \tag{4}$$

$$j_{\mathrm{H}_2,1} = \mathrm{L}_3 a_\mathrm{max} k_{\mathrm{m},\mathrm{H}_2} p_{\mathrm{H}_2} \tag{5}$$

$$j_{\mathrm{H}_2,3} = -f_1\left(c_\mathrm{A,near}\right) a_\mathrm{max} i_{0,\mathrm{H}_2} e^{-\alpha_{\mathrm{c},\mathrm{H}_2}\mathrm{L}_1\eta_{\mathrm{H}_2}} \tag{6}$$

$$j_{\mathrm{O}_2,1} = f_2\left(c_\mathrm{A,near}\right) a_\mathrm{max} i_{0,\mathrm{O}_2} \cdot \left[e^{\alpha_{\mathrm{a},\mathrm{O}_2}\mathrm{L}_2\eta_{\mathrm{O}_2}} - f_3\left(p_{\mathrm{O}_2}\right) e^{-\alpha_{\mathrm{c},\mathrm{O}_2}\mathrm{L}_2\eta_{\mathrm{O}_2}}\right] \tag{7}$$

$$j_{\mathrm{O}_2,3} = \mathrm{L}_4 a_\mathrm{max} k_{\mathrm{m},\mathrm{O}_2} p_{\mathrm{O}_2} \tag{8}$$

Due to the double layer capacity at the interface between the solid and the liquid phase an additional term has to be defined :

$$j_{\mathrm{dlc},1} = j_{\mathrm{dlc},3} = C_\mathrm{dl}\left(1-\mathcal{U}^\zeta\right)\frac{\partial \eta}{\partial t} \tag{9}$$

The total current density can thereby be calculated as

$$j = j_\mathrm{main} + j_{\mathrm{O}_2} + j_{\mathrm{H}_2} + j_\mathrm{dlc} \tag{10}$$

The overpotentials η, η_{O_2} and η_{H_2} are defined as

$$\eta_x = \phi_\mathrm{s} - \phi_\mathrm{l} - U_x \tag{11}$$

and the terminal voltage can be calculated by

$$U_\mathrm{cell} = \left.\phi_\mathrm{s}\right|_\mathrm{left} - \left.\phi_\mathrm{s}\right|_\mathrm{right} \tag{12}$$

Two further equations have to be considered expressing Ohm's law in the liquid phase

$$0 = \frac{\partial}{\partial x}\left(\mathrm{L}_5\frac{\partial \phi_\mathrm{l}}{\partial x}\right) + \frac{\partial}{\partial x}\left[\mathrm{L}_6\frac{\partial c_\mathrm{A,near}}{\partial x}\right] + j \tag{13}$$

and Ohm's law in the solid phase

$$0 = \frac{\partial}{\partial x}\left(\mathrm{L}_7\frac{\partial \phi_s}{\partial x}\right) - j \tag{14}$$

The acid concentration is calculated separately near the interface of the solid and liquid phase for a small constant volume $\varepsilon_\mathrm{l,near}$ and further away:

$$\varepsilon_\mathrm{l,near}\frac{\partial c_\mathrm{A,near}}{\partial t} = \mathrm{K}_5 j_\mathrm{main} + \mathrm{K}_6 j_{\mathrm{O}_2} + \mathrm{K}_7 j_{\mathrm{H}_2} + \mathrm{L}_8 k_\mathrm{near,far} \tag{15}$$

$$\frac{\partial \varepsilon_\mathrm{l,far} c_\mathrm{A,far}}{\partial t} = \frac{\partial}{\partial x}\left(\mathrm{L}_9\frac{\partial c_\mathrm{A}}{\partial x}\right) - \mathrm{L}_{10} k_\mathrm{near,far} \tag{16}$$

The local utilization of the active material $\mathcal{U}$ is defined by

$$\frac{\partial \mathcal{U}_1}{\partial t} = -\frac{j_\mathrm{main}}{Q_\mathrm{max}} \text{ and } \frac{\partial \mathcal{U}_3}{\partial t} = \frac{j_\mathrm{main}}{Q_\mathrm{max}} \tag{17}$$

The constants K_x only depend on the geometry of the cell whereas the coefficients L_x also depend on temperature, acid concentration and cell pressure. The values of the remaining coefficients are estimated by the parameter identification algorithm described below. For a more detailed description of the model the reader is referred to [1–5].

To solve this set of differential equations the control volume method as described in [3, 5, 8] is applied.

PARAMETER IDENTIFICATION

The parameter identification algorithm is used to estimate the initial values of 15 battery parameters, e.g. the utilizations $\mathcal{U}$ of the active materials, the fraction of liquid volume ε_1, the acid concentrations c_A and the double layer capacity C_dl. Some of the parameters are spatially distributed. Their initial values are however assumed constant within the regions of the cell. This is a valid simplification if the battery is carefully discharged with a low current to equalize the material distributions. A long rest period prior to the reference measurement ensures a constant acid concentration in the whole cell.

The cell voltage shows a highly nonlinear dependance on the model parameters. The error function, e.g. the sum of squared errors, contains many local minima that result in unacceptable identification results. Therefore deterministic local optimization algorithms, like the Gauss–Newton algorithm, are not well suited for the parameter identification [9]. Global stochastic optimization methods, like genetic algorithms, have shown to be capable of finding the global optimum of multimodal functions. However, the necessary population size for a good coverage of the parameter space results in a very high computational burden in combination with the complex and time–consuming simulation model.

With the proposed identification algorithm it is tried to reduce the number of simulations needed to obtain satisfactory results. Available expert knowledge about the lead–acid battery model is utilized during the identification process. The expert knowledge can be obtained from an analysis of the model equations and from experiments with the cell model. It is a qualitative description of the influence of some of the battery parameters on the voltage response of the model. The information can be used to calculate the optimal values of those parameters by means of an expert system. A fuzzy inference system embedded in an optimization loop is used for this purpose [10]. However, the influence of a number of parameters on the simulation results cannot be predicted. These parameters are treated with a GA. The resulting combination of fuzzy control and a stochastic optimization algorithm reduces the computational demands of the stochastic algorithm while maintaining its global search capabilities.

The structure of the combined identification method is shown in figure 2. Based on the current population, the genetic algorithm manipulates the individuals to generate a new population according to the principles of evolution [11]. This results in new combinations of the parameters with unpredictable behavior. The remaining parameters are adjusted with a control loop containing the fuzzy expert system. The difference between the simulated and measured voltage responses of the battery is resolved into several different quality criteria. They are calculated under consideration of the specific current test pattern used as excitation signal. Every criterion rates a certain aspect of the quality of the simulated parameter vector. The performance numbers are closely linked to the expert knowledge. The fuzzy inference system changes the battery parameters according to the calculated qualities and its rule base. This inner optimization loop is repeated until some stopping criterion is reached. It can be regarded as a control loop that tries to compensate the difference between measurement and simulation by adjusting the fuzzy–controlled parameters. After the inner loop is terminated, the next iteration of the genetic algorithm is launched.

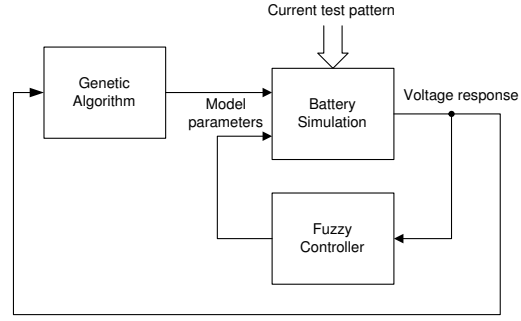

Figure 2: Block diagram of the parameter identification algorithm.

EXCITATION SIGNAL

A special current pattern was designed for the identification process. It is depicted in figure 3. The curve was carefully chosen to excite all states of the battery. It contains charge and discharge pulses of different durations and idle times in the whole range of the state of charge. The nonlinear relationship between cell voltage and current intensity is covered with different pulse amplitudes. The excitation signal can be used for lead–acid cells of different capacities by adapting the maximum cur-

rent intensity. The changes of the battery behavior with the state of charge are recorded by repeated application of the pulse pattern.

The measurement starts with a completely discharged battery. First a low charge current is applied for a longer time in order to reach a certain state of charge of the cell. After that the pulse pattern of figure 3 is applied. The first part consists of a set of 20 alternating charge and discharge pulses of short duration. The pulse time should be short enough that the effect on the acid concentration in the cell is negligible (e.g. in the range of 100 ms). The current intensity is increased with a constant ratio between adjacent pulses. The same pulse block is then applied one more time. After that comes another set of 20 alternating charge and discharge pulses, but with a longer duration,e.g. 5 seconds, and a longer idle time between them. The whole current pattern of figure 3 is subsequently repeated until the pressure in the battery reaches a predefined limit. Excessive gassing reactions begin after the cell is fully charged and cause the pressure rise. The measurement of the whole voltage response of a real cell may take up to 10 hours.

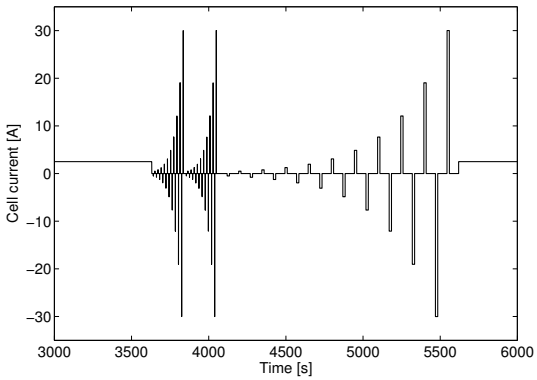

Figure 3: Current pattern used for battery identification.

The battery should be completely discharged before applying the test pattern. The discharge should be performed with a small current in order to equalize the material distributions within the cell. A rest period should follow after that to obtain a constant acid distribution. These steps are necessary to justify the simplifications made regarding the battery parameters.

Error function and quality criteria

A common choice for the error functions F of identification problems is the sum of some norm of the prediction errors [12]. For the present battery identification the sum of absolute values of the errors is considered,

$$F = \sum_{i=1}^{n} |U(t_\mathrm{i}) - U_\mathrm{M}(t_\mathrm{i})| \qquad (18)$$

with the voltage response of the model U_M and the measured cell voltage U. The sum is taken over all n discrete measurement samples. The simulated voltage is interpolated. The disadvantage of (18) is that the model responses to different excitations, like charge pulses, idle times and discharge pulses, are merged into one number. Some model parameters influence all the characteristic model responses. But there are also parameters that only act on specific characteristics. This information is lost by summing up the errors.

The fuzzy expert system uses more specific information to adjust its battery parameters. Distinct battery characteristics are required that are influenced by certain parameters. For that purpose 11 quality criteria Q_i are extracted from the simulated and measured voltage responses. Each of them is a measure of the difference between the real cell behavior and the model response. The criterion Q_1 is the voltage deviation at the low charge current between the pulse sequences, averaged over the whole state of charge. Q_3 describes the difference of the open circuit voltages, as depicted in figure 4. The value is averaged over all pulse sequences. Criterion Q_4 is the averaged difference in the increase of the open circuit voltage between subsequent pulse sequences.

The performance measure Q_5 is the difference of the heights of the voltage responses to a charge current pulse, as illustrated in figure 5. The value is averaged over all pulse sequences between the start of the measurements and the start of excessive gassing reactions. The increase in the voltage jumps between subsequent pulse sequences is assessed through criterion Q_7. When the utilization of the active cell materials approaches zero, a sharp rise in the cell voltage occurs even at low charge currents. The time of this rise is rated by quality criterion Q_9, as shown in figure 6.

The sum of absolute values of the quality characteristics is a measure of the model accuracy. However, only certain major effects can be sampled. Small deviations between real cell and model

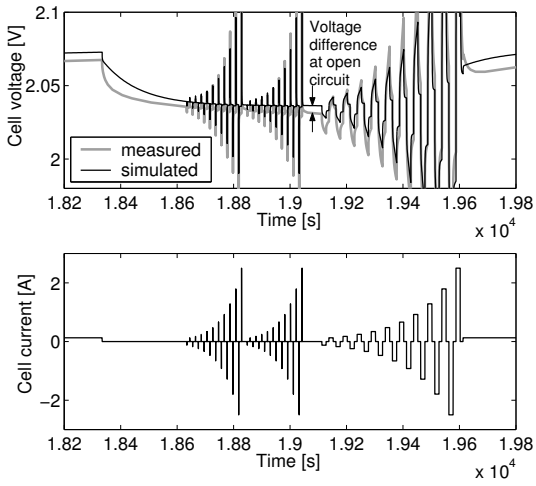

Figure 4: Quality criterion Q_3: Difference of the open circuit voltages.

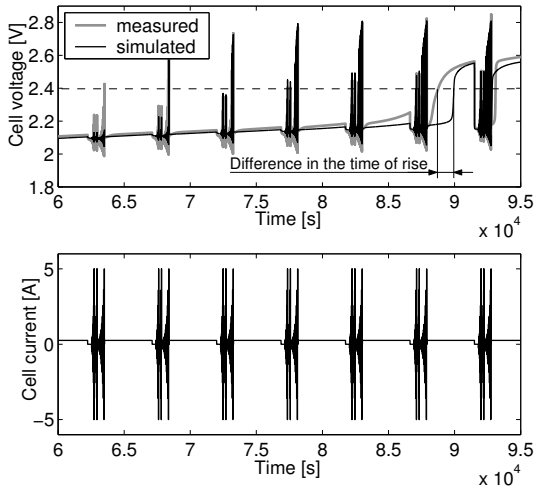

Figure 6: Quality criterion Q_9: Difference of the time of voltage rise when the battery is fully charged.

cannot be tracked with the quality measures. By using the sum of the qualities as error function, the emphasis can be put on identifying the most important battery characteristics. For the present approach a combination of the sum of absolute values of the prediction errors and the sum of quality measures is used:

$$F = w_1 \cdot \sum_{i=1}^{n} |u(t_\mathrm{i}) - u_\mathrm{M}(t_\mathrm{i})| + w_2 \cdot \sum_{j=1}^{k} Q_\mathrm{j} \quad (19)$$

The weights w_1 and w_2 determine the compromise between sampling the major effects only and evaluating the overall performance of the model. They are chosen to balance the contributions of the single terms.

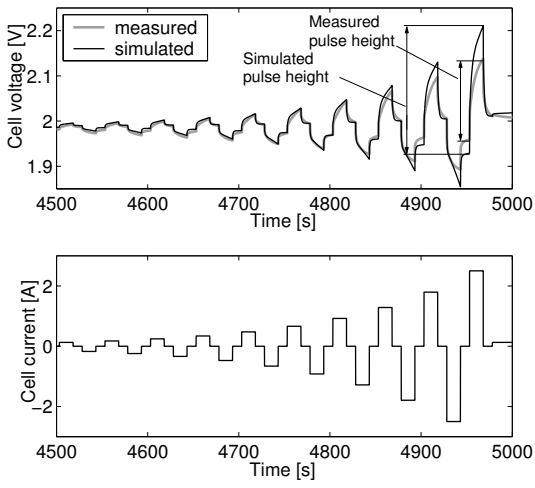

Figure 5: Quality criterion Q_5: Difference of the height of the voltage response to a charge current pulse.

Genetic Algorithm

A floating point GA [7, 11] is used as stochastic optimization strategy for the battery parameters with unpredictable behavior. The seven parameters ζ, $\varepsilon_{\mathrm{l,near}}$, $a_{\max} k_{\mathrm{PbSO_4}}$, $a_{\max} i_0$, $\alpha_{\mathrm{a,O_2}}$, $a_{\max} i_{0,\mathrm{O_2}}$ and $a_{\max} k_{\mathrm{m,O_2}}$ are identified with the GA.

For every individual, the GA calls the inner fuzzy optimization loop. The inner loop performs some iterations and returns the error function value of the individual. The fitness of the individuals is determined based on a linear ranking of the error function values (equation 19) of the population. The selection of a part of the population for recombination is made with stochastic universal sampling [11]. The recombination probability of the individuals is proportional to their fitness. The following recombination operation generates new individuals from the selected parents by intermediate crossover [7]. The parameters of the new individuals are mutated with a low probability. The mutation range follows a normal distribution. The fitness of the new individuals is calculated after invoking the inner optimization loop. They are afterwards reinserted into the population. The GA is based on an elitist strategy. The individuals with

the lowest fitness are always discarded in favor of the new ones. The next iteration is started with the selection until a stopping criterion causes the termination of the algorithm.

Fuzzy Expert System

Eight battery parameters are estimated with the fuzzy expert system. It adjusts the parameter values for a given set of GA optimization parameters to give the best possible model response [10]. The block diagram of this inner optimization loop is shown in figure 7. The battery parameters given by the GA are fed into the model and remain constant during the following steps. The initial values for the inner optimization parameters are taken from the individual of the preceding GA generation that is closest to the current individual. This is a kind of information sharing between the individuals of different generations. The battery simulation is performed with the initial parameter values in the first iteration. Afterwards the quality criterions are calculated from the voltage response. They are used as inputs of the fuzzy controller, which is implemented as a linguistic (Mamdani–type) fuzzy inference system [13]. Based on the expert knowledge formulated in rules, the controller produces output factors. The battery parameters are multiplicatively connected with the factors. The subsequent iteration is started with the modified fuzzy loop parameters. The inner loop is terminated if the prescribed maximum number of iterations is reached or if there is no significant improvement. The automatic stepwidth adaption is monitoring the actual and preceding values of the quality measures. If the fuzzy output factors show to be too conservative or too big, the stepwidths of the fuzzy outputs

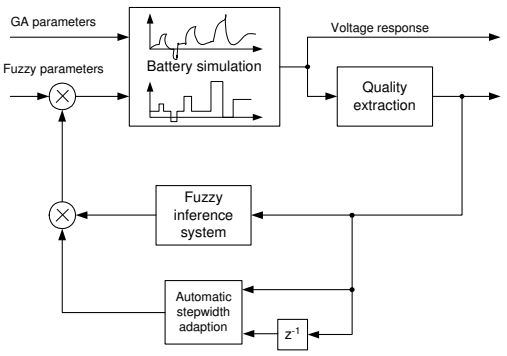

Figure 7: Block diagram of the internal fuzzy control loop.

are multiplicatively altered.

The input variables of the fuzzy system are normalized and then fuzzified with the three membership functions *negative*, *zero* and *positive*, as shown in figure 8. Gaussian and spline–based function definitions are used. Similarly, the fuzzy outputs consist of three membership functions *smaller*, *equal* and *bigger*. The minimum and maximum operators are used for the and–operation, implication and accumulation. Defuzzification is performed with the center of gravity method [13].

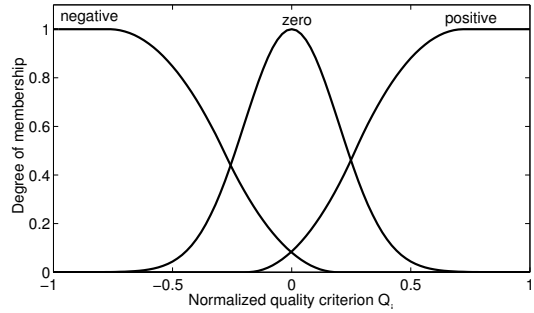

Figure 8: Membership functions for the linguistic variables Q_{i}.

The expert knowledge was accumulated by analyzing the underlying system of partial differential equations and performing simulation studies. It is a qualitative description of the influence of some battery parameters on the quality characteristics Q_{i} of the model output. A number of 33 rules was established for the fuzzy controller. They combine at most two input variables with two output factors. A few examples are:

IF Q_1 = *positive* AND Q_3 = *positive* THEN $k_{\mathrm{near,far}}$ = *equal* AND $c_{\mathrm{A,near}}$ = *smaller*

IF Q_1 = *zero* AND Q_3 = *negative* THEN $k_{\mathrm{near,far}}$ = *bigger* AND $c_{\mathrm{A,near}}$ = *bigger*

IF Q_9 = *positive* THEN $\mathcal{U}$ = *smaller*

IF Q_9 = *negative* THEN $\mathcal{U}$ = *bigger*

A demonstration of the performance of the fuzzy optimization loop is given in figure 9. The upper subplot compares the simulated voltage response after the first simulation run with the measured cell voltage. There is a distinct deviation of the simulation. The situation after the third iteration shows clear improvement, as depicted in the central subplot. After five calls of the simulation,

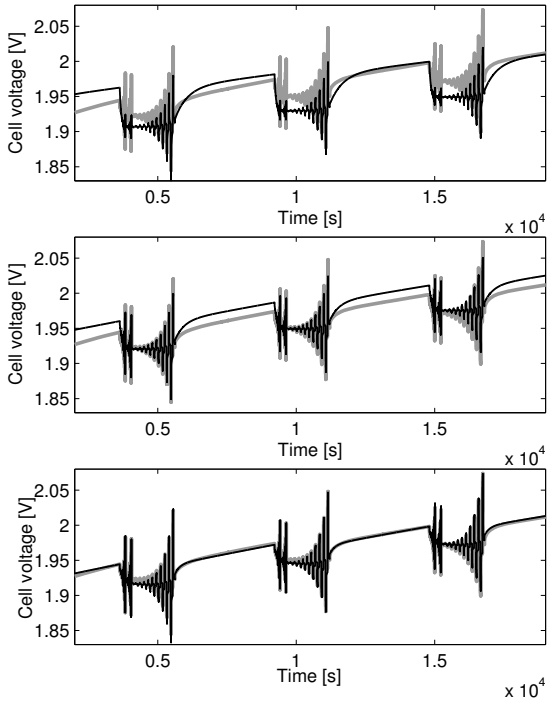

Figure 9: Performance of the internal fuzzy control loop: Simulated (black) and measured (gray) voltage responses after the first, third and fifth iteration.

the fuzzy control loop achieved a very good fit of the model parameters.

IDENTIFICATION RESULTS

The proposed identification algorithm was tested on several real lead–acid batteries. As an example, the identification results for a battery of the type Hawker Cyclon with a nominal capacity of $5\,\mathrm{Ah}$ are shown. The reference measurements were taken at an ambient temperature of $20°\mathrm{C}$. The proposed excitation signal was applied with a maximum charge and discharge current of $5\,\mathrm{A}$. The pulse width was set to $0.2\,\mathrm{s}$ for the short pulses and $15\,\mathrm{s}$ for the long pulses. The result was obtained after 20 generations of the GA with a population size of 45.

The complete reference curve of cell voltage and the identification result are depicted in figure 10. The response to a pulse sequence at a low state of charge of the cell is shown in detail in figure 11. The response at a high state of charge, where excessive gassing occurs, is illustrated in figure 12. There are some imperfections of the model, espe-

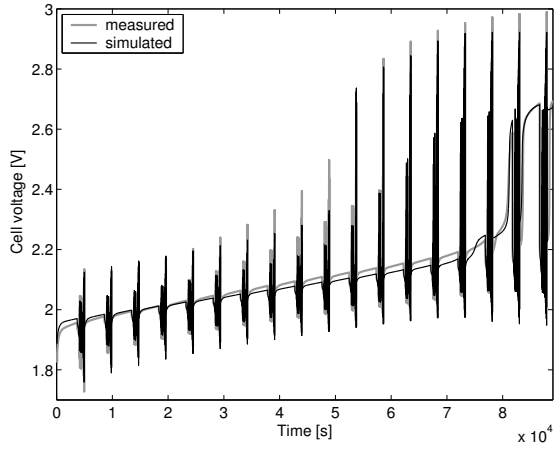

Figure 10: Identification result for a Hawker Cyclon 5Ah lead–acid cell.

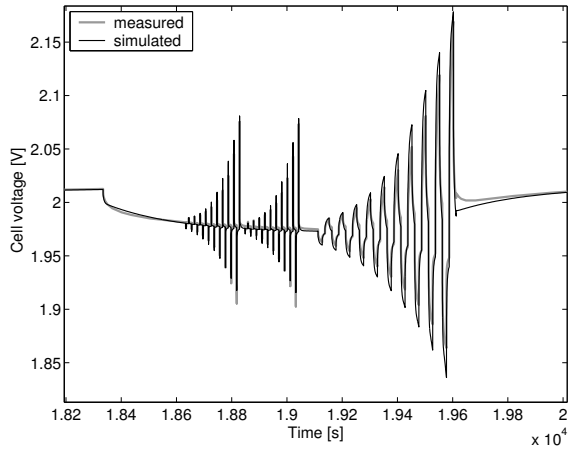

Figure 11: Detail of the identification result: Pulse sequence at low state of charge.

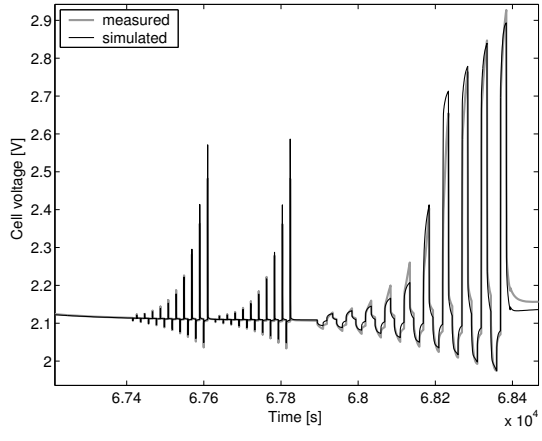

Figure 12: Detail of the identification result: Pulse sequence at high state of charge.

cially in the transient behavior. Nevertheless, the combination of the physical battery model and the proposed identification algorithm leads to a reasonable prediction of the cell behavior in the whole range of the state of charge.

CONCLUSION

An accurate lead–acid battery model consisting of a system of nonlinear partial differential equations was presented. It depends on a variety of parameters that cannot be measured. Periods of up to 15 hours may have to be simulated for parameter identification, which makes the solution of the forward problem quite time–consuming. Therefore a novel approach for the identification of the battery parameters was developed that integrates readily available expert knowledge regarding the influence of parameters on specific battery characteristics. These parameters are adjusted by a fuzzy controller containing the expert knowledge. The fuzzy control loop is invoked by a genetic algorithm that optimizes the unpredictable parameters. The presented results show that the novel method is able to provide good solutions. Due to the specialized expert knowledge, the method is only applicable to battery identification problems. Future research will be focussed on extending the method to a more general class of problems by automatic accumulation of expert knowledge from training data sets.

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